(Pages : 4) **S – 1308**

Reg. No. : ....

Name : ....

Ph.D. ENTRANCE EXAMINATION 2023

FACULTY OF SCIENCE

STATISTICS

Time : 3 Hours Max. Marks : 100

Instructions **:**

1) *Answer* ***any ten*** *questions each from* Section ***A*** *and Section* ***B.***
2) *Each question carries* **5** *marks.*
3) *No additional Answer sheets will be provided.*
4) *Candidates should clearly indicate the section, Question number in the answer booklet.*

Section – A

Research Methodology

Answer any **ten** questions. Each Question carries **5** marks.

1. What are the objectives and motivation of research?

2. Explain the technical difference between the terms- ‘Research Methods’ and ‘Research Methodology’.

3. What are the key characteristics of qualitative research? Comment on its strengths and limitations.

4. Describe the quantitative research methodology highlighting its positive and negative aspects.

5. Explain various steps in a research problem.

6. Describe the basic principles and relevance of research design.

7. What is literature review in scientific research? Elucidate its importance.

8. How do you develop a research plan? Present a short account of various steps involved in its development.

9. Explain the importance and need for research ethics.

10. Describe different methods of collecting review of literature leading to quality research output.

11. Explain the rules of writing references and bibliography.

12. Describe the procedure of writing thesis in Statistics.

13. Describe the creation of an efficient presentation, pointing out the aspects of organization delivery and communication in a presentation.

14. "Determination of sample size for a sample survey is an important problem of an experimental design." Suggest a method for determining the sample size.

15. Write short notes on — (a) Intellectual property rights and Patents and (b) Plagiarism in research.

(10 $\times$ 5 = 50 Marks)

Section – B

Statistics

Answer any **ten** questions. Each Question carries **5** marks.

1. Distinguish between 'finitely additive' and 'countably additive' set functions and show that a countably additive set function is finitely additive and continuous.

2. Define a measurable function. Let $\Omega = \{1,2,3,4\}$ and $\mathcal{F} = \{\phi, \Omega, \{1\}, \{2,3,4\}\}$. Is the function $f(x) = 1 + x$ a measurable function on Ω with respect to the σ-field $\mathcal{F}$? If not, give a non-constant function which is measurable with respect to the $\mathcal{F}$?

3. Define metric space and vector space and give an example to each with proper justification.

4. The joint probability density function of random variables X and Y is given by

$$f(x,y)=\begin{cases}2e^{-x}\,e^{-2y}, 0<x<\infty, 0<y<\infty\\ 0, \; otherwise\end{cases}$$

Compute (a) $P(X<Y)$ and (b) $P(X>1, y<1)$

5. (a) If X and y are two random variables, obtain the conditional expectation and conditional variance formulae involving them.

   (b) Mr. Basil forgot the 'four-digit password' necessary to log into his computer If he tries all possible password completely at random, discarding the Unsuccessful ones, what is the expected number of attempts to find the correct password? (3+2 = 5)

6. Define Hotelling's T^2 and Mahalanobis D^2 statistics. Bring out the relationship between them. What are the uses of these statistics?

7. Let (Ω, F, P) be the probability space, where $\Omega=\{1,2,3,...\}$ F is the power set of Ω and P is given by $P(n)=\dfrac{1}{n(n+1)}, n=1,2,....$

   Let $A_n=\{n, n+1,...\}$ Then show that $\sum_n P(A_n)=\infty$ but .$P(\lim Sup A_n)=0$.

   Is it a contradiction to the Borel-Cantelli Lemma? Comment.

8. Explain the different modes of convergence of sequence of random variables, stating the mutual implications between them.

9. Stating the relevant postulates, describe a Poisson process. If $\{N(t), t\geq 0\}$ is a Poisson process with rate parameter λ, obtain the correlation coefficient between $N(u)$ and $N(v), u, v\geq 0$.

10. What is uniformly minimum variance unbiased estimator (UMVUE)? Illustrate with an example, the method of obtaining the UMVUE using Rao-Blackwell theorem.

11. Let $X_1, X_2, \ldots, X_m$ be independent random sample of test scores by India in cricket matches played 'in India', assumed to be distributed as $N(\mu_1, \sigma_1^2)$. Let $Y_1, Y_2 \ldots, Y_N$ be another independent random sample of test scores by India for matches played 'in England', assumed to be distributed as $N(\mu_2, \sigma_2^2)$. It is desired to test the hypothesis that there is no difference in the score variabilities (ie., $\sigma_2^1 = \sigma_2^2$) against the alternative of different variabilities Develop a test procedure based on likelihood ratio test for large 'm and 'n'.

12. Present conceptual notes on —

    (a) Bayes' estimator,

    (b) Wald's SPRT, and

    (c) Empirical distribution function. (2+2+1 = 5)

13. Propose estimators of population mean, under simple random sampling without replacement (SRSWOR) and stratified random sampling. Compare the variances of the estimators for population mean under SRSWOR with stratified mean under proportional and optimal allocation.

14. Assuming a full rank linear model, $Y = X\beta + \varepsilon$, bring out the least square estimator and the best linear unbiased estimator of β.

15. Stating the linear model and the basic principles used in the analysis of data from Latin Square Design, give an outline of the conventional ANOVA table.

(10 $\times$ 5 = 50 Marks)