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Reg. No. : ....

Name : ....

Ph.D. ENTRANCE EXAMINATION 2023

FACULTY OF SCIENCE

MATHEMATICS

Time : 3 Hours Max. Marks : 100

Instructions **:**

1) *Answer* ***any ten*** *questions each from* Section ***A*** *and* ***B.***
2) *Each question carries* **5** *marks.*
3) *No additional Answer sheets will be provided.*
4) *Candidates should clearly indicate the section, Question number in the answer booklet.*

Section – A

Research Methodology

I. Answer any **ten** questions. Each question carries **5** marks.

1. Explain the research approaches.

2. What are the most important resources available for presenting mathematics?

3. Write short note on research and scientific method.

4. What are the objectives of research?

5. What is the difference between "research" and "rediscovery" ?

6. Explain the necessity of defining the problem.

7. What are the basic principles of experimental designs?

8. Explain the different research designs.

9. Briefly explain how to conduct a literature review during a research?

10. What are the precautions to be taken while writing a research paper?

11. What are the writing rules of mathematical research?

12. What is the difference between "theorem" and "conjucture"?

13. What is the difference between a "paper" and an "article"?

14. Explain the types of sampling designs.

15. What are the characteristics and functions of research paper?

($10 \times 5 = 50$ Marks)

Section – B

Mathematics

II. Answer any **ten** questions. Each question carries **5** marks.

1. Factorise $x^4 - 4$ into irreducible in $Z[x], Q[x], R[x]$ and $C[x]$.

2. (a) Show that if $n = u^2 + v^2$ is an odd integer then $n \equiv 1 \pmod 4$.

   (b) Show that the only positive integer solution to the equation $x^2 + 2 = y^3$ is $x = 5$, $y = 3$.

3. Prove that the L^p spaces are complete.

4. Prove that there is a continuous function that maps a Lebesgue measurable set to a non-measurable set.

5. Show that the equation $x^3 + y^3 + z^3 = 0$ has no solution in non-zero elements of $Z[\rho] = Z\left[\frac{-1+\sqrt{-3}}{2}\right]$. In particular show that it has no solution in non-zero integers.

6. Show that the linear mapping $f : R^3 \to R^3$ defined by $f(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$ is neither injective nor surjective.

7. Determine all elements of a ring that are both units and idempotent.

8. Prove that the function $f(x) = x^2 |\sin(1/x)|$ when $x \neq 0$ and $f(0) = 0$ is absolutely continuous.

9. Let $\sum a_n$ be an absolutely convergent series having sum s. Then prove that every rearrangement of $\sum a_n$ also converges absolutely and has sum s.

10. Prove that every open set S in R^n can be expressed as the union of a countable disjoint collection of bounded cubes whose closure is contained in S. Also deduce that S is measurable.

11. (a) Prove that $\pi^2 - 1$ is algebraic over $Q(\pi^3)$.

    (b) What is the smallest positive integer n such that there are two non-isomorphic groups of order n?

12. Prove that $G \oplus H$ is abelian if and only if G and H are abelian.

13. Determine all ring homomorphisms from Z to Z.

14. Prove that $Q(\sqrt{2}, \sqrt[3]{2}\ \sqrt[4]{2}, \ldots.)$ is an algebraic extension of Q but not a finite extension of Q.

15. (a) Define free group.

    (b) Prove that every group is a homomorphic image of a free group.

(10 $\times$ 5 = 50 Marks)