

Reg. No. :

Name :

Ph.D. ENTRANCE EXAMINATION, NOVEMBER 2022

FACULTY OF SCIENCE

MATHEMATICS

Time : 3 Hours

Max. Marks : 100

Instructions :

- 1) Answer **any ten** questions each from Section **A** and **B**.
- 2) Each question carries **5** marks.
- 3) No additional Answer sheets will be provided.
- 4) Candidates should clearly indicate the section, Question number in the answer booklet.

Section – A

Research Methodology

- I. Answer any **ten** questions All questions carry equal marks.
1. What is the difference between a thesis and a project?
 2. How do you create a good hypothesis?
 3. What are the objectives of research?
 4. What are the different methods of data collection?
 5. Distinguish between exploratory research and conclusive research.
 6. What is the basic principle of ANOVA?
 7. What are the features of a good research design?
 8. What are the important softwares used for typing Mathematics research papers?

9. What is the difference between a sample and a population?
10. What is the purpose of Mathematics Subject Classification (MSC)?
11. What are the characteristics and functions of a research paper?
12. What are the different steps involved in writing a research report?
13. Give a brief account of the use of computers in mathematical research.
14. What are the precautions to be taken while writing a research report?
15. Discuss the guidelines for preparing Bibliography.

(10 × 5 = 50 Marks)

Section – B

Mathematics

- II. Answer any **ten** questions. All questions carry equal marks.
1. If G is a group of order $11^2 \cdot 13^2$, find the number of 11-Sylow subgroups of G .
 2. Let X and Y be two normed linear spaces over \mathbb{R} or \mathbb{C} . Prove that if X is complete and Y is linearly homeomorphic to X , then Y is complete.
 3. (a) With usual notations define the family of functions \mathcal{R} . Prove that each function in $\mathbb{C} [a, b]$ possesses a best approximation in \mathcal{R} .
 (b) Prove that best approximations of the form $ax/(b|x| + c)$ do not exist for all functions in $\mathbb{C} [-1, 1]$.
 4. Using graphical method, solve the following Linear programming problem:
 Maximize $z = 3x + 2y$
 subject to,
 $-2x + 3y \leq 9$
 $3x - 2y \leq -20$
 $x, y \geq 0$.
 5. Solve the equation $x'' - \frac{2}{t}x' + \frac{2}{t^2}x = t \sin t, t \in [1, \infty)$ using the method of variation of parameters.

6. If F is the field of rational numbers, determine the degree of the splitting field of the polynomial $x^4 - 2$ over F .
7. Define the girth, circumference and diameter of a graph. Determine their values for the graph given in Fig 1

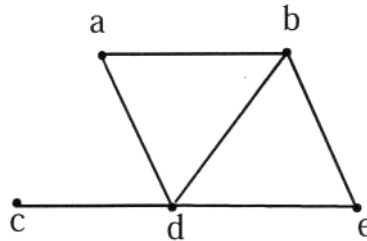


Fig. 1

8. Show that if μ is a non-negative set function on a ring, is countably additive and is finite on some set, then μ is a measure.
9. Define a path in a topological space X . Show that equivalence of paths is an equivalence relation on the set of paths in the space X .
10. Prove that every metric space is normal.
11. Consider a disk rolling on the horizontal xy -plane constrained to move so that the plane of the disk is always vertical. Derive the differential equations of constraint and show that they are nonholonomic.
12. Assume that α is monotonically increasing on $[a, b]$. Define upper Stieltjes integral of f and lower Stieltjes integral of f . Prove that $\underline{I}(f, \alpha) \leq \bar{I}(f, \alpha)$. Show by an example that the inequality can be strict.
13. Prove that for $p \geq 1$, $L^p(\mu)$ is a complete metric space.
14. Prove that every group of order 35 is cyclic.
15. Let A be a nilpotent 7 by 7 matrix with complex entries. Set $B = A - I$, where I is the 7 by 7 identity matrix. Determine the determinant of B .

(10 × 5 = 50 Marks)